Reliability Engineering

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Gyan Ranjan Biswal received his B.E. in Electronics Engineering from the Pt. Ravishankar Shukla University, India in 1999 and M. Tech. (Honors) in Instrumentation & Control Engineering from the Chhattisgarh Swami Vivekananda Technical University, India in 2009 followed by Ph.D. in Electrical Engineering, specialized in the area of Power System Instrumentation (Power Generation Automation) from the Indian Institute of Technology Roorkee, India in 2013.

He is expertise in Design and Development of cooling systems for large size electrical generators, and the C&I of process industries. He has been in academia for about twelve years. Presently, he is with VSS University of Technology, Burla, India at the capacity of Associate Professor, EEE from Dec. 2016, and HOD, EEE from Jan. 2020 to Feb. 2023, and conferred with the Best faculty Award for the AY 2021-22 under Professor/ Associate Professor category . He has more than 75 publications in various Journals and Conferences of Internationally repute to his credit. He also holds a patent as well, and filed one more. He also adapted one international edition book published by Pearson India. He received research grants of US $80,000 (INR 64 lakhs). He has been supervised 02 Ph.D. theses and 09 Masters' theses, and ongoing 03 PhD theses. He has also been recognized with many national and international awards by elite bodies. He has been awarded with CICS award under the head of Indian National Science Academy for travel support to USA, MHRD Fellowship by Govt. of India, and Gopabandhu Das Scholarship in his career. His major areas of interests are Power System Instrumentation, Industrial Automation, Robust and Intelligent Control, the Smart Sensors, IoT enabled Smart Sensors, the Smart Grid, Hydrogen Cooling System, Hydrogen Storage and Its Processing, Fuel Cell lead Sustainable Sources of Energy, and System Reliability.

Dr. Biswal is a Fellow IE (India), Senior Member of IEEE, USA, and Life Member of ISTE, India. He is actively involved in review panels of different societies of international repute viz. IEEE, IFAC, and the ISA. Currently, he is also actively involved as a Member of IEEE–SA (Standards Association) working groups; IEEE P1876 WG, IEEE P21451-001 WG, and IEEE P1415. He has also been invited for delivering guest lectures at World Congress on Sustainable Technologies (WCST) Conf. 2012, London, UK, INDICON 2015, New Delhi, India, National Power Training Institute (NPTI), Nangal, India, and G.B. Pant Engineering College, Pauri, Gharwal, India, Surendra Sai University of Technology, Burla, as a guest expert in 2016 IEEE PES General Meeting Boston, MA, USA, 1st Annual Webinar of Complex Engineering System, Politecnico di Milano, Italy in 2022, and Keynote lecture in 12th EAI International Conference on Sensor Systems and Software, Portugal in 2021.

Syllabus

Reliability Engineering

MODULE-I (6 HOURS)

Types of System, Qualitative and Quantitative assessment, Use of quantitative assessment, Reliability Definition and Concepts, Reliability Indices and Criteria, Reliability and Availability, Absolute and Relative Reliability, Reliability Evaluation Technique, Reliability Improvement, Reliability Activities in System Design & its Economics, Basic Probability Theory, Binomial Distribution and its engineering applications.

MODULE-II (10 HOURS)

Network modeling concepts, Series & Parallel Systems, Series-Parallel System, Partially Redundant & Standby redundant System. Modeling and Evaluation Concept, Conditional Probability Approach, Cut Set Method, Application and Comparison of Previous Technique, Tie Set Method, Connection Matrix Technique, Event Trees, Fault Tree, Multi-Failure Mode.

MODULE-III (8 HOURS)

Distribution Concept & terminologies, General Reliability Function& their evaluation techniques, Shape of Reliability Function. The Poison Distribution & the Normal Concept, Exponential, Weibull, Gamma, Rayleigh, Lognormal and rectangular distributions, Data Analysis, System Reliability Evaluation of different kinds of Using Probability Distributions, Mean Time to Failure, Wear out And Component Reliability, Maintenance And Component Reliability.

MODULE-IV (8 HOURS)

Discrete Markov Chains: General Modeling Concept, Stochastic Transitional Probability Matrix, Time Dependent Probability Evaluation, Limiting State Probability Evaluation, Absorbing States, Application of Discrete Markov Technique.

Continuous Markov Process: General Modeling Concept, State Space Diagrams, Stochastic Transitional Probability Matrix, Evaluating Limiting State Probabilities, Evaluating Time Dependent State Probabilities, Reliability Evaluation in Repairable System, Mean Time to Failure, Application of Technique To Complex System.

MODULE-V (7 HOURS)

Frequency andDuration Technique: Application to Multistate Problems, Frequency Balance Approach, Two Stage Repair and Installation Process. Approximate System Reliability Evaluation. System with Non-Exponential Distribution. Monte Carlo Simulation.

Text and Reference Books

Recommended Text Books:

1. Roy Billinton, Ronald N. Allan. "Reliability Evaluation of Engineering Systems"Second Edition.

Reference Books:

* Gupta A.K., Reliability, Maintenance and Safety Engineering. University Science Press.

Other Important References

Reference Sites:	
1.	NPTEL, The National Programme on Technology Enhanced Learning (NPTEL): https://nptel.ac.in/
2.	MIT OpenCourseWare : https://ocw.mit.edu/index.htm
3.	https://www.youtube.com/channel/UC0ISZ4dMZcIBeIzjZVRZhJw/videos [Gyan Ranjan Biswal @gyanranjanbiswal5649]

Course Outcomes

Upon successful completion of this course, you (students) will be able to

CO1	Define the basic terms in reliability engineering concepts.
CO2	Construct and implement the network modelling of simple and complex systems.
CO3	Evaluate probability distribution for reliability of a system.
CO4	Incorporate discrete and continuous Markov processes for reliability evaluation.
CO5	Express competence on approximate reliability evaluation techniques.

* # Therefore, it is required to ~~optimize the process model dedicated for plant unit(s)~~ optimize ~~the~~ no. of components / redundancy required at each stage of process model for achieving optimized performance of the system with minimal investment / cost to set up the plant units(s). ~~It is equally known as optimizing~~ Such mechanism of optimizing the process model dedicated for specific plant unit(s) known as **process optimization**.

It is referred for intelligent selection of ~~redundancy~~ stage-by-stage redundancy to ~~work out achieve~~ work out optimized relationship betn cost & ~~efficiency~~ throughput of the system.

System Reliability

1

Fault Tree Analysis (Fault tree analysis)

Requirements of redundancy:

* There are two approaches ~~of~~ for improving the reliability of the system: fault avoidance and fault tolerance. Fault avoidance is achieved by using high-quality and high-reliability components and is less expensive.

However, no one component provides ~~the~~ 100% reliability. Even though in practice the reliability of components are assumed ~~assumed~~ ~~taken~~ 20% less from max. life claimed by component reliability data sheet or (~~10% less~~ atleast 10% less from threshold value) as per safety norms by industrial std. of practices. ~~and therefore fault tolerance~~. As fault tolerance is achieved by redundancy, which proportionally increases design complexity & cost, [ ] [If it becomes apparent that the systems reliability will not be adequate to meet the desired goal at the specified mission duration (operation / process duration), steps ~~are~~ required be taken to determine the best way to improve the systems reliability so that it will reach the desired target.]

(* max. achievable reliability is 99.9%)

Therefore reliability optimization is required to select intelligently the no. of redundant component required at specific stage of process in a plant / factory to improve the system reliability and that should be most efficiently is most effectively.

** (PhD / RTA / Engg Reliability) ← Soft copy

Simulation MATLAB ! do

(1) Prepare equi S^3RS – HCS fault tree using $\mathbb{R}$;
Fig. 10.13 (p. 280)

(2) Use Chapter 10 to prepare fault tree diagram with proper event symbol used in flowchart.

(3) Subdiagram: Ex. is given of 2-out-of-3 RBD at (p. 296) Fig. 10.35 – 37.

\+

Fig. 10.41 (p. 300)

* ours is 1-out-of-2 & 1-out-of-3

(4) ~~Rather~~ Replace present Fig. 1 in paper by RBD as like Fig. 10.41 + Equi fault tree diagram

* Our work is to compare the proposed process model with existing systems/ model. ~~by~~ ~~which~~ in terms of system reliability; When component reliability are taken in each model are same.

Now, in order to take decision that how many components reliability need to improve, so that overall system reliability could be improved by desired margin (R_s), ~~must~~ required to deal with cost.

Cost doesn't ~~mean~~ necessarily have to be dollars. It could be non-monetary resources, such as time.

★ By associating cost values to the reliabilities of the system's components, ~~we can find an~~ it could be find an "optimum design" that will provide the required reliability at a minimum cost.

The cost function of the reliability for each component must be quantified ~~before~~ to avoid needlessly expenses on overdesign. However, there are many cases where no such information is available. For this reason, a general (default) model of cost vs. the components reliability was developed for performing reliability optimization.

cost funct^n

$$C_i = f(R_i) \quad \text{——— (1)}$$

Reliability of each component

$$C_i(R_i) = e^{(1-f)\cdot\frac{R_i - R_{min,i}}{R_{max,i} - R_i}} \quad \text{——— (2)}$$

min & max value of the optimization to be performed.

penalty (cost) funct^n of component reliability.

dimensionless

feasibility / cost index of improving component reliability relative to other component of system.

Imp ~~Limitation of improving component reliability w.r.t. cost to improve system reliability~~

* In general,

improvements in individual/ selected component's reliability exponentially ↑es the cost ~~w.r.t. improvements in reliability~~ i.e. become is not feasible all the time.

Maximum Achievable Reliability

~~by Dep~~

* The costs near the max. achievable reliability are very high and the actual value for the max. reliability is usually dictated by technological / financial constraints.

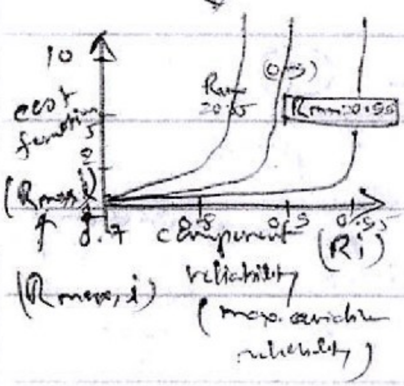
Note: Almost, any component can achieve a very high reliability value, provided the mission time is short enough.

Ex: A component with an exponential distribution

- of one failure/hour has R(t) drops below 1%. for mission greater than (>5) hours

- however R(t) = 99.9% if mission hour is < 4 sec

Need of improve System Reliability
OR
restriction of component reliability



However, such assumptions/ cases can not be acceptable, in case of process control and monitoring of (any) process station component; where model (group of specific and dedicated components in the order) is involved to process for the system. As process control and monitoring is an continuous (on-going) event.

Therefore, such restrictions failed in such cases.

$$\left.\begin{aligned} C_s(R_s) &= C_1(R_1) + C_2(R_2) + \cdots + C_n(R_n), \\ i &= 1, 2, \ldots n \end{aligned}\right\}$$

cost of system

Ex: $0.9 = R_1 \cdot R_2 \cdot R_3$

$\Rightarrow C_s(R_s) = C_1(R_1) + C_2(R_2) + C_3(R_3)$

5. **Optimization:** The optimization for a system of three (3) units in series will be different then the optimization for a system consisting of same 3 (three) units in ||el.

The optimization ~~for a sys~~ occurs by varying the reliability values of the components within ~~their~~ their respective constraints of max. & min reliability in a way that the overall system goal is achieved.

* Max. achievable reliability is 99.9%.

Implementing the optimization

5 * our research area

Reliability optimization routine

Fault tolerance	Fault avoidance
→ Improvement in reliability is achieved by adding (in ‖el) extra redundant component.	→ focus on improving component reliability (life). ~~by~~
→ Limitation: Identical component ~~required~~ ↑es (double) the budget in respect to the component in some cases; but in few cases it is cost-effective also.	→ improvement in reliability of component occurs linearly; but cost ↑es exponentially.

5-α Weibull's distribution :

$$R(t) = e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$

mission duration (in hours)

$\beta \leftarrow$ shape parameter (unitless)

scale parameter of distribution (in hours)

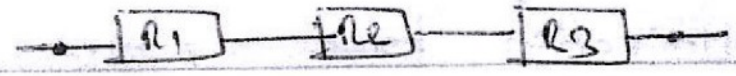
$$\left(\begin{array}{c}\beta > 1 \\ t < \eta\end{array}\right)$$

$R_{S3RS}(t)$
$R_{Hong}(t)$
$R_{Brooks}(t)$
$R_{Death}(t)$

— i.e if $R(t)$, t, β known then η_i could be find out for each model corresponds to given system reliability.

$R_1 = 70\% = 0.7$, $R_2 = 0.8$, $R_3 = 0.9$

Ex:-



$t = 100$ hours

$\eta = 312$ hours

Weibull Distribution:

1 Let,

$$R_s(t) = 0.90 = R_1(t) \cdot R_2(t) \cdot R_3(t) \quad \text{— (1)}$$

$$\therefore \; R_{1,2,3} = e^{-\left(\frac{t}{\eta}\right)^{\beta}} \quad \text{— (2)}$$

and

$$C_T \text{ (cost funct}^n\text{)} = C_1(R_1) + C_2(R_2) + C_3(R_3) \quad \text{— (3)}$$

$$C_{1,2,3}(R_{1,2,3}) = e^{(1-f)\frac{R_i - R_{min,i}}{R_{max,i} - R_i}} \quad \text{— (4)}$$

in this case, if

$$R_{max\ 1,2,3} = 0.999$$

then $R_{min\ 1,2,3} = ?$

if $t = 100$ hrs, $\eta = 312$ hrs & $\beta = 1.318$

$$R_{min,1,2,3} = e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$

$$= e^{-\left(\frac{100}{312}\right)^{1.318}}$$

$$\therefore R_{min\ 1,2,3} = 0.79995 \quad \rightarrow \underline{\text{Ans}}$$

also

$f \rightarrow$ is the feasibility (cost index) of improving a components reliability relative to the other components in the system;

$$f = \left(1 - \frac{5}{10}\right) = 0.5$$

(2)

say

$$R_{oi}(t=100) = 0.9655$$

overall reliability

$$= e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$

$$= e^{-\left(\frac{100}{\eta}\right)^{1.318}}$$

$$\Rightarrow \underline{\underline{(\eta_{\beta=1.4})}}\ \eta_{oi} = 1269.48 \text{ hours.}$$

Range of improvement :

* The difference betn the initial reliability of a component and ~~is~~ its maximum ~~reliability~~ achievable reliability is called the range of improvement for that component.

And, the greater this difference, the greater the cost of improving the reliability of a particular component relative to the other component involves in model.

"Note": Reliability growth models such as the Crow (AMSAA), Duane, Gompertz and Logistic models can be used to describe the "cost as a function of reliability".

* How (redundancy) fault tolerance method contributes to ↓es the finished product cost?

Ans:-

Total product cost (annual / per day) = Operation cost + Plant/Unit installation cost + Maintenance cost

(OC) + (IC) + (MC)

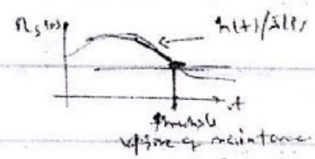
Now, if redundancy is there, then

OC ↓es → as active duration/ plant working hours ↑es

↓

as maintenance duration ↓es

→ as $R_s(t)$ ↑es →



∴ productivity per hour/day ↑es

↓

& In that way operation cost (OC) ↓es.

While IC is ↑ one time

~~& duration of IC~~

⇓

overall "Total cost ↓es"

Note: Component reliability is an impotant issue in the field of reliability engg. However, system reliability of a plant model; does matter, whenever purpose is to design and implement a system. Thus, it is equally important in case of huge and complex system like automation of power plant/station as a component of power system.

Overall improvement in system reliability of model ~~effect~~ correlates as follows:

System reliability ↑

↕

maintainability ↓

↕

Sleeping time / maintainance duration ↓

↕

throughput / process duration ↑

or

active mode of plant.

↕

efficiency of the system ↑

as plant spends more time in active mode.

↓ cost per hour (rating)

as productivity is high @ hours.

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I.

HCS (1-out-of-2) strategies

Patents.

	S^3RS - HCS	Hargreaves et al	Blatter et al	Brosnihan et al
$R_s(t)$				
$R_s(t)$				
η_{0}, availability				
MTTR = MTBF = $\bar{T}$ = mean				
MTTR				
σ (std. dev)				
t_{med}				
t_{mode}				
t_d				
TR				

* Major goal :

① System reliability $\rightarrow R_s(t)$

② FTA ; $F_s(t) = 1 - R_s(t)$

II. H2 based Power Plant

As process control & monitoring ~~plant function(s) software~~ is

system ~~of both one server~~ based on score algorithm

apart from capacity (2-out-of-3) redundancy.

* T-83A (2×8) or ~~(2×10)~~ (1×16)

1-out-of-2

2-out-of-3

as is better than

1-out-of-3

3-out-of-5

if further capacity need to ↑es.

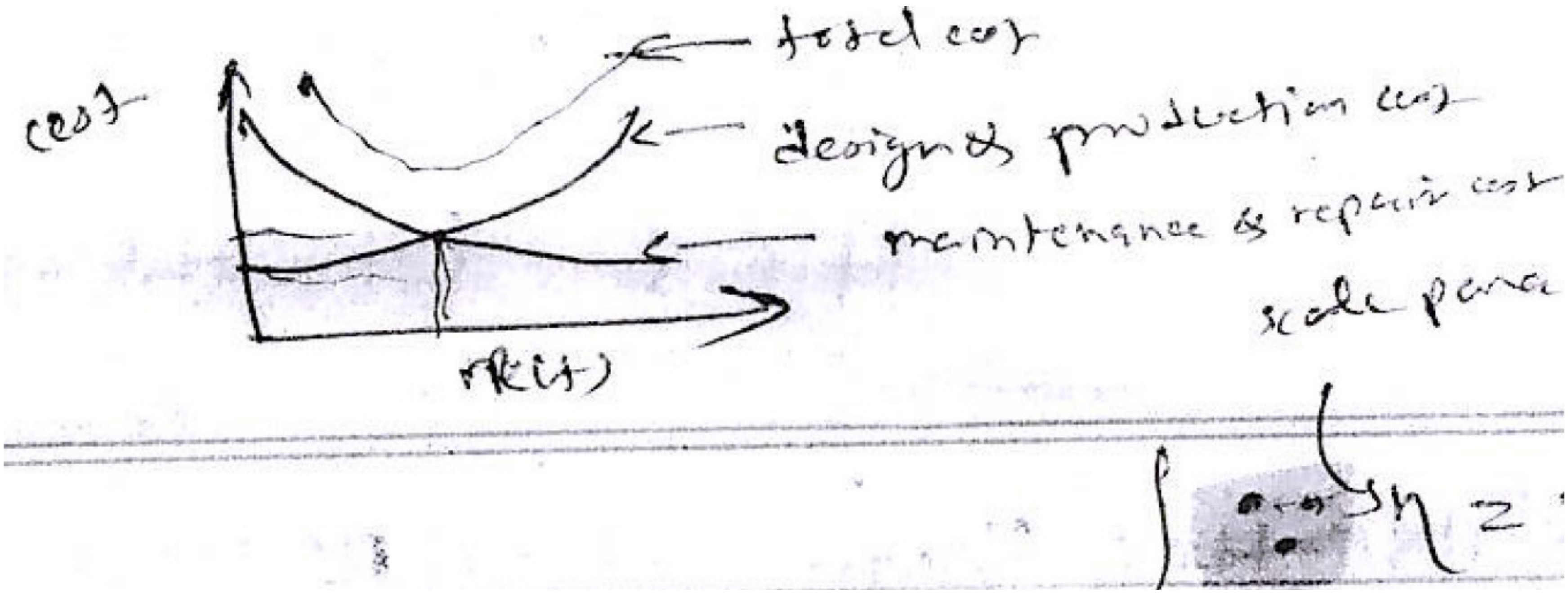
then (2 × 2-out-of-3).

III: CEP:

2-out-of-3 for 2×500 MW
or
2×600 MW plant
with structural change in design.

+

1-out-of-2 go for (120 – 300 MW).


total cost
design & production cost
maintenance & repair cost
scale para
cost
R(t)

take
$t = 100$ ~~days~~ days
$\eta = 312$ days
$\beta = 1.318$

Weibull

$\eta = 365 - 53 = 312$ days
$t =$
$\beta = 1.318 - 1.4$
shape para.

$$R(t) = R_{min} = e^{-\left(\frac{t}{\eta}\right)^{\beta}} = \exp\left[-n\left(\frac{t}{\eta}\right)^{\beta}\right]$$

ans.

$$\eta_{o,i} = 1$$

$$R_s(t) = \text{any MRT}$$

$$F_s(t) = 1 - R_s(t)$$

$$f(t) = -\frac{dR(t)}{dt} = \left(\frac{\beta}{\eta}\right)\left(\frac{t}{\eta}\right)^{\beta-1} \cdot \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right]$$

$$* \quad MTTF = \eta\, \Gamma\left(1+\tfrac{1}{\beta}\right) = \int_0^\infty R(t)\,dt \quad \because f(t) = 1 - R(t)$$

$$\sigma^2 = \eta^2 \left\{ \Gamma\left(1+\tfrac{2}{\beta}\right) - \left[\Gamma\left(1+\tfrac{1}{\beta}\right)\right]^2 \right\}$$

Std. $= \sqrt{\sigma^2}$

$$= \left[\int_0^\infty t^2 f(t)\,dt - (MTTF)^2\right]$$

$$t_{med} = \eta\left(-\ln(R)\right)^{1/\beta}$$

$$t_{mode} = \begin{cases} \eta\left(1-\tfrac{1}{\beta}\right)^{1/\beta}, & \because \beta > 1 \\ 0, & \text{for } \beta \le 1 \end{cases}$$

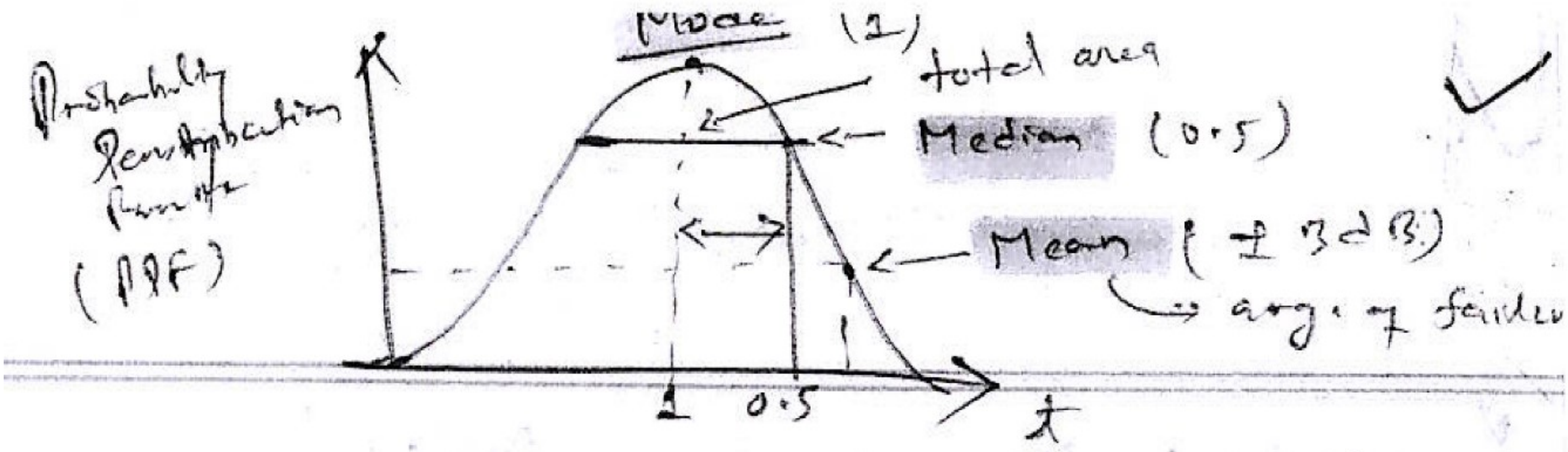
Also

② MTTR in terms $H(t)$ OR $\lambda(t)$

↳ hazard rate.

$$MTTR = \int_0^\infty t\, h(t)\, dt = \int_0^t h(t')\, dt'$$

$$= \int_0^\infty [1 - H(t)]\, dt$$



also

$$H(t) = 1 - e^{-t_{rmed}/MTTR}$$

if $t = t_{med}$

$\therefore$ $t_{med} = -MTTR \ln[1 - H(t)]$

note

$R(t_{median}) = 0.5$

otherwise

$$H(t) = 1 - e^{-t/MTTR}$$

and

(Mean Time To Failure) $\underline{MTTF} = \int_0^\infty R(t)\,dt$

* K - out - of n Redundancy

$$R(s) = \sum_{x \geq k}^{n} \binom{n}{x} R^x (1-R)^{n-x} = \sum_{x \geq k}^{n} P(x)$$

$\Rightarrow R_s(t) = 1 - [1 - R(t)]^n$

Ex) CMFR = 0.6

$R(t) = 0.4$

$R_s(t) = 0.95$

$n = \frac{\ln(0.05)}{\ln(0.6)} = 5.86 \simeq 6$ redundancy required

required to calculate at each stage

7 HCS (s3RS & all other)

* Take 'Γ' values from Text we I refer in M.Tech Reliability Engg. — refer notes to get author detail or phone to (Anil Mishra)

KPI (Key perform index)

① Availability = $\frac{\text{Operating time (OT)}}{\text{OT} + \text{MD}}$ × 100 % = $\frac{\text{up time}}{\text{up time} + \text{down time}}$

MD → Maintenance delays

also, Availability % = $\frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}}$

(Typical avg : 40 – 60 %)

② ∴ MTBF = $\frac{\text{Actual OT}}{\text{No. of MD}}$ = MTTF = $\bar{T} = \int_0^\infty R_s(t)\,dt$

(Typical value avg. around 1 hours)

③ ∴ MTTR = $\frac{\text{Actual } \cancel{\text{OT}} \text{ delay time}}{\text{No. of MD}}$

Typical value is ≃ 20 min

IITR – central library

A: System reliability : Page, Alain ; Gondran Michel ; Griffin, E. (620.0045 P12S)

B. Reliability in engg. design: Kapur, K.C. ; Lamberson, L.R. (620.0045 K12R)

C. Introduction to reliability and maintainability engineering : Ebeling, Charles E. (620.00452 E201)

D. Handbook of reliability engineering : Pham, Hoang (620.00452 P47H)

E. Reliability engineering : Srinath, L. S.
(620.00452 S66R)

F. Reliability engineering : Balaguruswamy, E
(620.00452 B16R)

G. Solid State Devices : Benn G. Streetman

H. Process Control : Bequette (PHI)

I. Process Control Instrumentation Technology : Curtis Johnson (PHI)

Weibull Distribution.

Ex:

$$R(t) = e^{-(t/\eta)^{\beta}} = e^{-\left(\frac{100}{800}\right)^{0.6}} = 0.75 \Rightarrow R(100)$$

S3RS ⇒ $R_{max}(t) = ? = e^{-\left(\frac{?}{312}\right)^{1.318}} = 0.90$ — ①

Hon $= ? =$ $= 0.$

Blat.

Brown

$t = ?$

$\eta = 312$ day (given)

$\beta = 1.318 - 1.4$ (given)

$R(t) = 0.90$ (S3RS)

$t_{0.1} = ?$

* after burn in period

$$R\left(\frac{t}{T_0}\right) = e^{-\left[\left(\frac{t+T_0}{\eta}\right)^{\beta} + \left(\frac{T_0}{\eta}\right)^{\beta}\right]}$$

$T_0 \rightarrow$ burn-in period = 200 day let, if $t = 100$ day

(take double of t)

= if $t = 100$, $T_0 = 200$

then $R\left(\frac{100}{200}\right) = 0.857$

$\leftarrow 0.90$ (say)

$$t_d = -\eta \times [\ln \cdot R(t)]^{1/\beta}$$

$$= -312 \times [\ln(0.9)]^{1/1.318}$$

$=$

Weibull p.d.f. (also density fun) $= f(x) = a x^b \exp\left(-\frac{a x^{b+1}}{b+1}\right); x \geq 0$

& distribution fun $= F(x) = 1 - \exp\left(-\frac{a x^{b+1}}{b+1}\right); x \geq 0$

$a \rightarrow$ scale parameter

$b \rightarrow$ shape parameter

✓ **Algo development**

Paper 4, 5: (J. + Int. Conf.)

Filling → Fl_T83B, Fl_T83C

Feeding → Fd_T83B, Fd_T83C

Pronunciation → P_T83B, P_T83C

DePronunciation → DP_T83B, DP_T83C / D_T83C

* Also try to simulate at different ~~test~~ range

(1) Ex: Heat exchanger $\rightarrow \{0.0 - 0.8\}$

&

Valve $\rightarrow \{0.5 \text{ to } 0.8\}$

(2) $R_{TA} = R_{TB} = R_{TC} = \{0.0 \text{ to } 0.9\}$

&

$\{0.3 \text{ to } 0.9\}$

(3) STVL $\rightarrow \{0.0 \text{ to } 0.95\}$

&

$\{0.7 \text{ to } 0.95\}$

* For control action of CNT → PVS3B/C.

$$\begin{cases} f(x) = \begin{cases} 0\,; & x \neq x_1 \\ 1\,; & x = x_1 \end{cases} & \text{—— (1)} \\ \lim_{n \to 0} f(x) = f(1+n) = f(1-n) = f(1) & \text{—— (2)} \end{cases}$$

te
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E. Balagurusamy

① **Note :** $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t}\, dt = (\alpha-1)!$

$= \lfloor(\alpha-1)$

$\therefore \Gamma(1+1/\beta) = \int_0^\infty t^{1+1/\beta-1} e^{-t}\, dt = 1/\beta\,! = \lfloor\beta^{-1}$

2

Nonlinear Hazard Model (Weibull distribution model)

$$Z(t) = a t^{b} = h(t) \quad \text{——— (1)}$$

(general form of hazard model)

$$R(t) = \exp\left[-a t^{b+1}/(b+1)\right] \quad \text{——— (2)}$$

$$f(t) = a t^{b} \exp\left[-a t^{b+1}/(b+1)\right] \quad \text{——— (3a)}$$

$$= a t^{b} \times R(t)$$

$$\boxed{f(t) = h(t) \times R(t)} \quad \text{——— (3b)}$$

$a \rightarrow$ scale parameter / η
$b \rightarrow$ shape parameter / β

$\Rightarrow$ in form of η, β

$$R(t) = \exp\left[-\frac{\eta \cdot t^{\beta+1}}{(\beta+1)}\right] \qquad \text{—— } (2')$$

$$h(t) = \eta t^{\beta} \qquad \text{—— } (1')$$

and

$$f(t) = h(t) \times R(t) \qquad \text{—— } (3')$$

$$MTTF = \frac{\Gamma\left(\frac{1}{(b+1)}\right)}{(b+1)\left[\frac{a}{(b+1)}\right]^{1/(b+1)}} = \frac{\Gamma\left(\frac{1}{(\beta+1)}\right)}{(\beta+1)\left[\frac{\eta}{(\beta+1)}\right]^{\frac{1}{(\beta+1)}}}$$

$$MTTF = \frac{d\,\Gamma(d)}{(a d)^{d}} \quad \text{———} \quad (4)$$

$$\because d = \frac{1}{\beta+1} = \frac{1}{b+1}$$

System Reliability Models

I. Systems with components in Series

$$MTTF = \int_0^\infty R(t)\,dt = \frac{1}{\sum_{i=1}^{n} \lambda_i} = \frac{1}{\sum_{i=1}^{n} \frac{1}{T_i}}$$

$\because$ System failure rate

$$\lambda_s = \sum_{i=1}^{n} \lambda_i$$

$$\Rightarrow \boxed{h_s = \sum_{i=1}^{n} h_i} \quad \text{--- (1)}$$

$$\therefore \frac{1}{T} = \frac{1}{T_1} + \frac{1}{T_2} + \frac{1}{T_3} + \dots + \frac{1}{T_n}$$

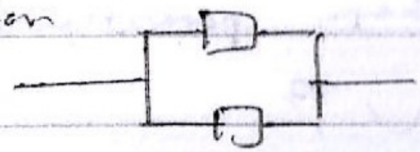
II. Systems with ||el components

With ~~const. failure rate (CFR)~~ ⇒

~~MTTF~~

- reliability of each unit ⇒ $p = 1 - Q^{1/m}$

∴ For



⇒ $R = 2p - p^2$

⇒ $R = 0.96$ i.e. if $p = 0.8$

⇒ $R = 0.99$ $\quad$ $p = 0.9$

$$\boxed{R_{m+1}(t) = 1 - [1 - p(t)]^{m+1}}$$

system rel

reliability of single comp

no. of components in ||el

∴ $\Delta R_m(t) = R_{m+1}(t) - R_m(t)$

$= p(t)\,[1 - p(t)]^m$

with CRR ⇒

$$R(t) = 1 - [1 - e^{-\lambda t}]^m$$

$$MTTF = \frac{1}{\lambda} \sum_{i=1}^{m} 1/i$$

III. K-out-of-m System

reliability

$$P(m, x) = B(m, x)\, p^x (1-p)^{m-x} \quad \text{--- (1)}$$

Binomial coefficient

$$B(m, x) = \binom{m}{x}$$

$$\therefore R = \sum_{i=k}^{m} B(m,i)\, p^{i} (1-p)^{m-i} \quad \text{——— (2)}^{*}$$

For CFR

$$R(t) = \sum_{i=k}^{m} B(m,i)\, e^{-i\lambda t} \left[1-e^{-\lambda t}\right]^{m-i} \quad \text{——— (3)}$$

Ex:-

** For 2-out-of 3 system

each component
- has
$p = 0.9$
i.e. CFR

$$R = B(3,2)\, p^2 (1-p)^{3-2} + B(3,3)\, p^3$$

$$= B(3,2)\, p^2 (1-p) + B(3,3)\, p^3$$

$$= 3p^2 - 3p^3 + p^3$$

$$= p^2 (3 - 2p)$$

$$\therefore R = 0.972 \text{ for } p = 0.9$$

also

$$MTTF = \int_0^\infty R(t)\, dt = \lim_{s \to 0} R(s) = \frac{1}{\lambda} \sum_{i=k}^{n} \frac{1}{i}$$

* System with mixed-mode failure

as our S3RS-HCS case

* Note: For simplifying FTA eqn, assume ~~identical~~ common reliability of identical elements.

E. Balagurusamy : "Chap. 6"

"Redundancy Techniques in System Design."

* Various approaches are:

1. System / Unit redundancy : simplest & most straightforward.
   i.e. duplicate path of entire system/unit itself.

2. Component redundancy : redundant path ~~of~~ for each components.

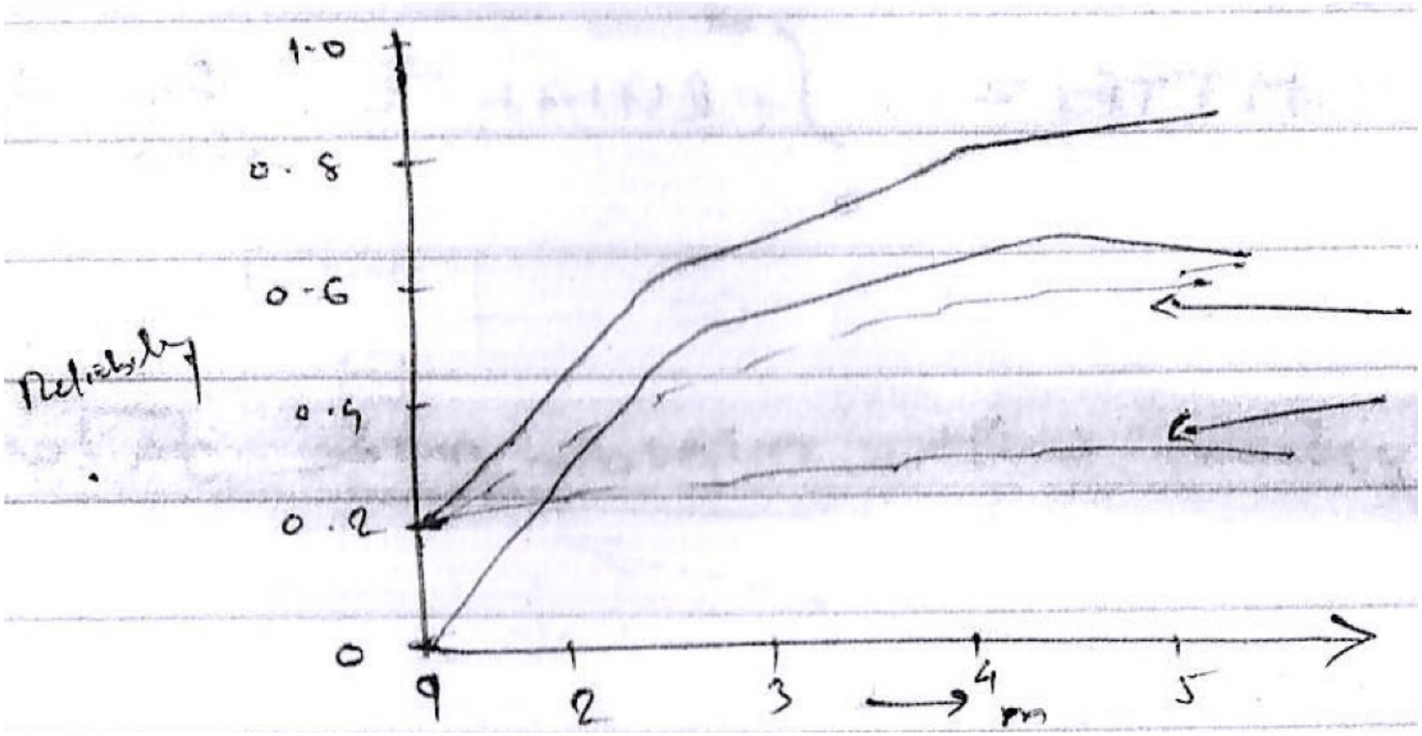
presently exist

3. Reliability & Cost - optimization : i.e weak component should be identified & strengthen to ↑es the reliability of system

4. Mixed redundancy : mix 1, 2, 3 depending upon configuration & reliability requirements.

* goal

This approach is choosing mixed ~~re~~ redundancy.



- - - - unit redundancy

—— component "

$$R_C = \left[1 - (1-p)^m\right]^n \quad \text{——— (1)}$$

$$R_U = 1 - \left[(1-p^n)\right]^m \quad \text{——— (2)}$$

. Other techniques are

1. Weakest - link technique : wherever required, provide redundancy at that only.

2. Mixed redundancy

3. Standby Redundancy

5* M Imp Part ← S3RS-H CS

Standby redundancy

(I) simple Standby System.

→ In standby redundancy the failed component is replaced by a standby unit through action of switch/ contact which is actuated by feedback sensing & control device.

Note : Not all components / equipments are suitable for active redundancy or parallel redundancy.

For e.g. Rs, Cs create design problems when they are put actively in ||el. The failure of ~~on~~ one of two such components operating in ||el will change ckt - constant ($\tau = R_{eq} C_{eq}$).

Similarly, two electric generators of diffn freq. cannot be ~~||el~~ put in ||el ~~together~~ ||el together.

→ such case require Standby redundancy.

$$\boxed{R = p_1 p_2 p_c + (1-p_1) p_2 p_c p_s + p_1 (1-p_2) p_t p_c} \quad \text{(1)}$$

- p_1 — reliability of Unit-1
- p_2 — " " Unit-2
- p_c — " of switch (contact)
- p_s — " " sensing & control device
- p_t — probability of an inadvertent & premature switching may not occur

assuming that $P_t = 0$

$$R = P_1 P_2 P_c + (1 - P_1) P_e P_c P_s + 0$$

$$R = P_c (P_1 + P_2 - P_1 P_2) \quad \text{--- (2)}$$

if $P_1 = P_2$

$$\boxed{R = P_c P (2 - P)} \quad \text{--- (3)}$$

so the increase in reliability due to redundancy

is $R - P$, i.e

$$[P_c(2-P)-1].P \quad \text{--- (4)}$$

& it will be positive only when

$$\boxed{P_c > \frac{1}{2-P}} \quad \text{--- (5)}$$

Generalized

* let $(m-1)$ units are standby to support one basic unit

(r out of m) standby redundancy

governed by possion density func" : $f(x) = \dfrac{(\lambda t)^x}{\lfloor x} \cdot e^{-\lambda t}$ (~~governed by~~ —— (1)

$\because x \rightarrow$ probability of failure

$\therefore R(t) = \sum_{x=0}^{m-1} f(x) = e^{-\lambda t} \sum_{x=0}^{m-1} \dfrac{(\lambda t)^x}{\lfloor x}$ —— (2)

∴ Based on Markov-model / Joint-density-functⁿ technique: For a two-unit system,

$$R(t) = e^{-\lambda t}(1+\lambda t) \longrightarrow (3)$$

$$R_{stand} = p\left[1 - \ln(p)\right] \longrightarrow (4)$$

$$R_{active,2} = p(2-p) \longrightarrow (5)$$

$$\therefore \; R_{stand} - R_{active,2} = p\left[p - 1 - \ln(p)\right]$$

$$= p\left[p - 1 + \lambda t\right]$$

$$\therefore P = e^{-\lambda t} = 1 - \lambda t + \frac{(\lambda t)^2}{\lfloor 2} - \frac{(\lambda t)^3}{\lfloor 3} + \cdots$$

$$\therefore R_{stand} - R_{act} = P\left[\frac{(\lambda t)^2}{\lfloor 2} - \frac{(\lambda t)^3}{\lfloor 3} + \cdots\right]$$

Interest / Scopy

** The Weibull Distribution

one of most widely used lifetime distributions in reliability engineering. It is a versatile distribution that can be take on the characteristics of other types of distributions, based on the value of the ~~shape~~ shape parameter (β).

1. The Three-parameter Weibull Distribution

* pdf: $$f(T) = \frac{\beta}{\eta}\left(\frac{T-\gamma}{\eta}\right)^{\beta-1} e^{-\left(\frac{T-\gamma}{\eta}\right)^{\beta}} \quad \text{— (1)}$$

$\because \quad f(T) \geqslant 0, \quad T \geqslant 0$

or

$\gamma, \beta > 0, \quad \eta > 0, \quad -\infty < \gamma < \infty$

and

η = scale parameter

β = shape/ slope parameter

γ = location parameter

2. The Two parameter Weibull Distribution

by setting $\gamma = 0$;

* pdf : $f(T) = \dfrac{\beta}{\eta}\left(\dfrac{T}{\eta}\right)^{\beta-1} \cdot e^{-\left(\frac{T}{\eta}\right)^{\beta}}$.

[illegible]

$t = T$

$$\left\{\begin{array}{l} \Rightarrow f(T) = \lambda(T) \cdot R(T) \longrightarrow (2) \\ \because R(T) = e^{-\left(\frac{T}{\eta}\right)^{\beta}} \ \& \\ \lambda(T) = \dfrac{\beta}{\eta}\left(\dfrac{T}{\eta}\right)^{\beta-1} \end{array}\right.$$

3. The One parameter (η) Weibull Distribution

by setting $\gamma = 0$ &

assuming $\beta = C$ = constant / assumed value

$$f(T) = \frac{C}{\eta}\left(\frac{T}{\eta}\right)^{C-1} \cdot e^{-\left(\frac{T}{\eta}\right)^{C}}$$

* here η is only unknown parameter. Here we assume 3
that β is known priori from past exp. fm identical products.
The advantage of doing this is that data sets with few / no failures can be analysed.

** Weibull Statistical properties

4. The Mean or MTTF $(\bar{T})$

The Mean, $\bar{T}$ also called MTTF / MTBF,

$$\boxed{\bar{T} = MTTF = MTBF = \gamma + \eta \cdot \underbrace{\Gamma}_{\text{gamma function}}\left(\frac{1}{\beta}+1\right)} = \int_0^\infty T \cdot f(T)\, dT \qquad (4)$$

$\because$ $\Gamma\left(\frac{1}{\beta}+1\right)$ is the gamma functn evaluated by value $\left(\frac{1}{\beta}+1\right)$.

$$* \because \boxed{\Gamma(n) = \int_0^\infty e^{-x}\, x^{n-1}\, dx} \quad \text{——— (5)}$$

Ex

$$\Gamma(1/\beta+1) = \int_0^\infty e^{-x} x^{1/\beta+1-1}\, dx$$

$$= \int_0^\infty e^{-x} x^{1/\beta}\, dx$$

Alsandro Birolini

$\because \beta = 1.318$, $\Gamma(1/\beta+1) = \cancel{1.75}\ \Gamma(1.75)$

$= 0.915$

* Two parameter

$$\boxed{\bar{T} = \eta \cdot \Gamma\left(\frac{1}{\beta}+1\right)} \quad \text{—— (6)}$$

$\left(\because \beta \geq 0, \text{ Two parameter} \right)$

&

$$\left.\begin{aligned} \eta = \bar{T} &= MTTR = MTBF, \because \beta = 1 \\ &= \Gamma\left(\frac{1}{1}+1\right) = \Gamma(2) = 1 \end{aligned}\right\}$$

* A shape para. (β) is any para. of a probability distribution that is neither a location para (γ) nor a scale parameter (η) (nor a functn of either or both of these only, such as a rate parameter). Such a para. (β) affect the shape of a distribution rather than simply shifting it (as a ~~loc~~ γ does) or ~~stretching/shrinking it (as η does).~~

5. The Median ($\check{T}$)

$$\boxed{\check{T} = \gamma + \eta (\ln 2)^{1/\beta}} \quad \text{--- (7)}$$

For Two, parameters $\gamma = 0$;

$$* \quad \boxed{\check{T} = \eta (\ln 2)^{1/\beta}} \quad \text{--- (8)}$$

$$\left(\because \int_0^{\check{T}} f(t)\, dt = 0.5 \right)$$

6. The Mode / Modal life ($\tilde{T}$)

$$\frac{d[f(t)]}{dt} = 0$$

i.e

$$\boxed{\tilde{T} = \gamma + \eta\left(1 - 1/\beta\right)^{1/\beta}} \quad \text{—— (9)}$$

for Two parameter

$$\boxed{\tilde{T} = \eta\left(1 - 1/\beta\right)^{1/\beta}} \quad \text{—— (10)}$$

7. The Standard Deviation (σ_T).

$$\left\{ \begin{aligned} \sigma_T &= \eta \cdot \sqrt{\Gamma\left(\frac{2}{\beta}+1\right) - \Gamma\left(\frac{1}{\beta}+1\right)^2} \quad \text{—— (11)} \\ \sigma_T &= \sqrt{\eta^2 \cdot \Gamma\left(\frac{2}{\beta}+1\right) - (MTTF)^2} \end{aligned} \right.$$

EXTRA

① Weibull

cumulative density function, cdf:

$$F(T) = 1 - e^{-\left(\frac{T-\gamma}{\eta}\right)^{\beta}}$$; Three parameter

$$F(T) = 1 - e^{-\left(\frac{T}{\eta}\right)^{\beta}}$$; Two "

—— (12)

②

$$R(T) = e^{-\left(\frac{T-\gamma}{\eta}\right)^{\beta}}$$; Three parameter

$$R(T) = e^{-\left(\frac{T}{\eta}\right)^{\beta}}$$; Two parameter

—— (13)

(3) The Weibull Conditional Reliability Function

$$R(t/T) = \frac{R(T+t)}{R(T)} = \frac{e^{-\left(\frac{T+t-\gamma}{\eta}\right)^{\beta}}}{e^{-\left(\frac{T-\gamma}{\eta}\right)^{\beta}}}$$

or

$$R(t/T) = e^{-\left[\left(\frac{T+t-\gamma}{\eta}\right)^{\beta} - \left(\frac{T-\gamma}{\eta}\right)^{\beta}\right]} \qquad (14)$$

④ The Weibull Reliable life

$$T_R = \gamma + \eta \cdot \left\{ -\ln \left[R(T_R) \right] \right\}^{1/\beta} \quad \text{--- (15)}$$

(starting the mission at age zero)

for Two parameters ; $\gamma = 0$

$$\boxed{T_R = \eta \left\{ -\ln \left[R(T_R) \right] \right\}^{1/\beta}} \quad \text{--- (16)}$$

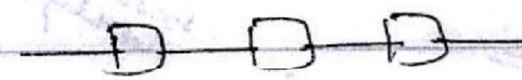
(5) The Weibull Failure Rate Function

$$\lambda(T) = \frac{f(T)}{R(T)} = \frac{\beta}{\eta}\left(\frac{T-\gamma}{\eta}\right)^{\beta-1} \quad \text{(17)}$$

B. S. Dhillon

"Reliability Evaluation of Two-state Device Networks"

1. Series Network



$$R_i(t) = e^{-\int_0^t \lambda_i(t)\,dt}$$

$$R_s(t) = e^{-\lambda_1 t} \cdot e^{-\lambda_2 t} \cdots e^{-\lambda_n t}$$

$$MTTF = \int_0^\infty R(t)\,dt = \sum_{i=1}^{n} \lambda_i^{-1}$$

② Parallel Network

$$R_p(t) = 1 - \prod_{i=1}^{k} F_i(t)$$

$$R_p(t) = 1 - \prod_{i=1}^{k} (1 - e^{-\lambda_i})$$

$$MTTF = \frac{1}{\lambda} + \frac{1}{2\lambda} + \frac{1}{3\lambda} + \cdots + \frac{1}{k\lambda}$$

$$R_p(t) = e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t}$$

$$MTTF = \int_0^\infty R_p(t) = \frac{(\lambda_1+\lambda_2)}{\lambda_1\lambda_2} - \frac{1}{\lambda_1+\lambda_2}$$

3

k-out-of-m Unit

is generally weibull distribution

$$R_S(t) = R_{k/m}(t) = \sum_{i=k}^{m} \binom{m}{i} [R(t)]^i [1 - R(t)]^{k-i}$$

Ex: 2-out-of-4

$$R_{2/4}(t) = 3e^{-4\lambda t} - 8e^{-3\lambda t} + 6e^{-2\lambda t}$$

$\because t = 100$

$$R_{2/4}(100) = 0.5252$$

Hot / Warm

★ 4 **Standby Redundant System**

Imp ★

system Reliability

$$R_s(t) = \sum_{i=0}^{k} \left\{ \int_0^t \lambda(t) \cdot dt \right\}^i e^{-\int_0^t \lambda(t)\, dt} (\lfloor i)^{-1} \quad \text{--- (1)}$$

most surely in exam

Ex:- for $k = 2$, $\lambda(t) = 0.003$ failure/hour

MTTF = ?

#
$$\boxed{R_s(t) = \sum_{i=0}^{2} \frac{(\lambda t)^i e^{-\lambda t}}{\underline{|i}}} \quad \text{--- (2)}$$

$$\therefore MTTF = \int_0^\infty R_s(t)\, dt$$

$$\boxed{MTTF = \frac{3}{\lambda}} = \frac{3}{0.003} = 1000 \text{ hours}$$

Ans

{ * In our case:

$\lambda = h$ = hazard rate = 0.005 failure/day

"Reliability Determination Methods"

1. Network Reduction Techniques (NRT) pp. 32
   - Tie set (min)
   - Cut set (min)
   - ★ (benefit of both "technique")
2. Path - Tracing Techniques pp. 34
3. Decomposition Techniques pp. 35
4. Minimal Cut Set Techniques pp. 37
5. Delta - Star Method pp. 38
6. Markov Modeling (CT & DS) pp. 43
   - CT: Continuous Time
   - DS: Discrete States
7. Binomial Method pp. 44

" Fault Trees " ← pp. 81 – 94.

★ pp. 81 – 83 : Symbols & Definitions

VImp ★ pp. 173 : System Safety Analysis Classification

① Conceptual : PHA, PSMA

② System Design : Subsystem hazard
System "
Safety Logic
FHA, FEA

③ Functional system : OHA, OSMA

Functional Hazard Analysis

★ "A Generator Unit Model with PM"

Preventive Maintenance

(pp. 296 – 298)

availability

$$\boxed{A_g = \frac{\mu_1 \mu_2}{k_1 k_2}} = \lim_{t \to \infty} P_0(t)$$

repair rate

Alessandro Birolini

" Reliability Engineering "

5*

(1) Detailed in pp. 544, Table A9.6.

Ex: $\Gamma(z) = \int_0^\infty x^{z-1} e^{-x} dx$

$Re(z) > 0$, Euler integral.

$\Gamma(z+1) = z\Gamma(z)$ with

$\Gamma(1) = 1$

Special values : $\Gamma(0) = \infty$,

$\Gamma(1/2) = \sqrt{\pi}$,

$\Gamma(1) = \Gamma(2) = 1$,

$\Gamma(\infty) = \infty$.

(2) Special contents :

pp. 31 — Reliability block dia & functn.

* (3) Concept of redundancy (pp. 43)

- → (i) Active / Parallel, hot
- → (ii) Warm / lightly loaded
- → (iii) Standby / Cold, unloaded

4 → 9 Failure Mode Analysis (pp. 73)

* b Reliability aspect in Design Review (pp. 78)

- → FMEA / FMECA
- → FTA *
- → Ishikawa Diagram / Fishbone Diagram
- → K-T Method
- → Pareto Diagram
- → Correlation Diagram

"Fault Tree Analysis (FTA)"

(Paper 4)

* (1) $R(T) = e^{-\left(\frac{T}{\eta}\right)^{\beta}}$
* $h(T) = \frac{\beta}{\eta}\left(\frac{T}{\eta}\right)^{\beta-1}$
* $\therefore f(T) = h(T) \times R(T)$
* $F_s(T) = 1 - R_s(T)$ * $R_s(T) = 1 - [1 - R(T)]^n$

$\because T = 100$ days

$\eta = 312$ days

$\beta = 1.318$

Target ⇒

$T = 105$

$\eta = 312$ days

$\beta = 2.118$

$R_s(t) ?$

$R_{s3Rs}(t) \approx 90\% = 0.9047 / 0.91, T = 105$ days

$R_{Haorg}(t) \approx 0.576, T = 236$ days.

$R_{beat}(t) \approx \cancel{0.8299\ 0.79}\ 0.8259 = 142$ days

$R_{Brag}(t) \approx 0.648, T = 210$ days.

$R_{Adel}(t) \approx 0.5995, T = 227$ days

(2) $\bar{T}$ = Mean = MTTF = MTBF

$$= \eta \cdot \Gamma\left(\frac{1}{\beta}+1\right) = \int_0^\infty T \cdot f(T)\, dT$$

$= \eta$ (eta) × gamma function $\left(\frac{1}{\beta}+1\right)$

beta

$\because \beta = 1.318,\ 1.718,\ 2.118$

$\Rightarrow$ MTTF = MTBF = $312 \times \Gamma(1.75)$

* $\check{T} = \text{Median} = \eta \times (\ln 2)^{1/\beta}$

$$\left(\because \int_0^{\check{T}} f(T)\,dT = 0.5 \right)$$

* & $\tilde{T}$ = Mode = Modal life

$$= \eta\left(1-\frac{1}{\beta}\right)^{1/\beta}$$

$$\left(\because \frac{d f(t)}{dt} = 0\right)$$

∴ * **Note:** for ~~constant~~ fixed values of β (slope para), ~~other~~ η (scale parameter (Max. O.T.)) will be changed, as $\bar{T}$, $\check{T}$, $\tilde{T}$ & σ of all four models/system is different.

as T is different of all.

& η → represents max. operating duration

$$R = e^{-\left(\frac{T}{\eta}\right)^{\beta}}$$

$$\ln(R) = \left(-\frac{T}{\eta}\right)^{\beta} \Rightarrow \{\ln(R)\}^{1/\beta} = -\left(\frac{T}{\eta}\right)$$

$$\Rightarrow \boxed{T = -\eta \times \{\ln(R)\}^{1/\beta}}$$

* To workout T; always find out T at R [illegible] 21.318 (std. value)

③ σ = Std. deviation for ~~[illegible]~~ Reliability (R)

$$= \eta \cdot \sqrt{\Gamma\left(\frac{2}{\beta}+1\right) - \Gamma\left(\frac{1}{\beta}+1\right)^2}$$

$$= \sqrt{\eta^2 \cdot \Gamma\left(\frac{2}{\beta}+1\right) - (MTTF)^2}$$

(4) The **Weibull Reliable life** (T_R)

$$T_R = \eta \times \left\{ -\ln\left[R(T_R)\right] \right\}^{1/\beta}$$

if $R(T_R) = 0.5$ then $\because R(T_R) = 0.5$

$T_R = \check{T} = \text{Median}$ i.e. $T_R / \check{T} \neq$ Median

$$MTTR = \int_0^\infty [1 - h(T)]\, dT.$$

$\because$ $$h(T) = 1 - e^{-T/MTTR}$$

or

$$t_{med} = -MTTR \ln[1 - h(T)]$$

$$MTTR = \frac{t_{med}}{\ln[1-h(T)]}$$

Note

(a) $$MTTR = \frac{\text{Actual delay time}}{\text{No. of Maintenance delay (MD)}}$$

(Typical $\approx$ 20 min)

(b) $$\eta_{oi} = \text{Availability} = \frac{OT \leftarrow \text{operation time}}{OT + MD} \times 100\%.$$

(Typical $\approx$ 40 – 60%)

w.r.t. 24 hrs.

(c) $$MTBF = MTTF \doteq \bar{T} = \frac{\text{Actual OT}}{\text{No. of MD}}$$

(Typical $\approx$ 1 hours)

IV. Analysis Framework

Since modeling is an extensive and resource consuming task it is of high importance that the tool, in this case the PRM, does support the modeler's needs with precision. The PRM presented in this paper is a modification of the ArchiMate metamodel [22]. Entities irrelevant to the field of SA have been removed, and appropriate attributes and attribute relations describing availability have been added.

* In section IV. FT

A. *Defining Availability*

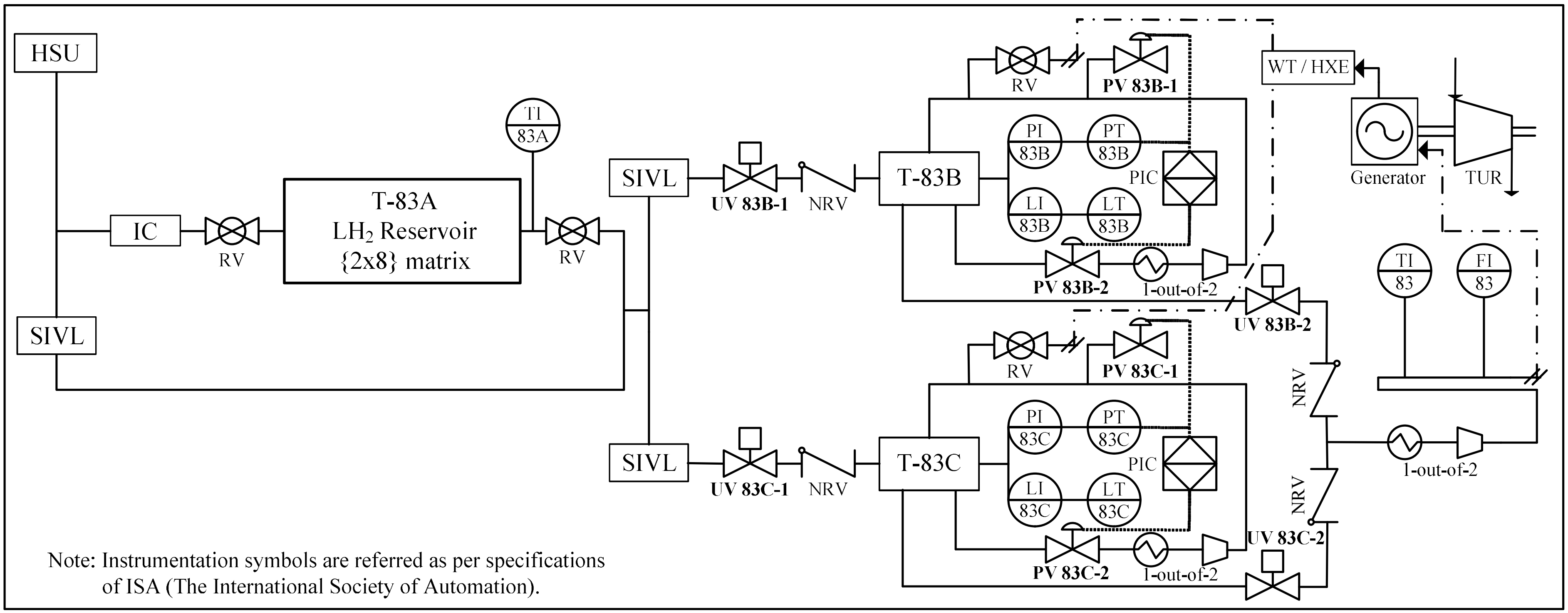
* → like this defining h(t), MTTF and TR in such case cont — FTA chapter

The attribute of availability is, in accordance with [23] defined as the proportion of time a system, or a system component, is expected to be available during the operation window. Availability is assessed through the variables of Mean Time To Failure (MTTF) and Mean Time To Repair (MTTR):

$$Availability = \frac{MTTF}{MTTF + MTTR}$$

According to the equation, availability of a system or a system component can either be improved by reducing the mean time to failure or improving the mean time to repair.

‘Six Stage Redundant Structure’–HCS



Schematic Diagram

❖ G. R. Biswal, R. P. Maheshwari, and M. L. Dewal, “Modeling, Control and Monitoring of S[3]RS based Hydrogen Cooling System in Thermal Power Plant,” *IEEE Trans. Industrial Electronics*, Vol. 59, No. 1, pp. 562-570, Jan. 2012.

- Increased system reliability;

- Lesser chances of failure ;

- Minimum time for maintenance (Weibull reliable life), T_R and T_{Rmin},

- Improved mean time to repair (MTTR), and

- Lesser hazard rate as against the existing systems, indicate that the process life of the proposed models are longer than that of existing systems.

- G. R. Biswal, R. P. Maheshwari, and M. L. Dewal, "Modeling, Control and Monitoring of S^3RS based Hydrogen Cooling System in Thermal Power Plant," *IEEE Trans. Industrial Electronics*, Vol. 59, No. 1, pp. 562-570, Jan. 2012.

System Reliability of the Existing versus the Proposed HCS

Systems	**Symbol**	System Reliability	
		Redundancy function of the system	**(in percent)**
Hargrove et al.	$\Re 1$	$R_X \cdot R_{TB} \cdot R_{HE1} \cdot R_{HE2}$	57.60
Krützfeldt and Musil	$\Re 2$	$R_{TA} \cdot R_{TB} \cdot R_{HE1} \cdot R_{\gamma}$	69.11
Blatter et al.	$\Re 3$	$R_{TA} \cdot R_{TB} \cdot R_{\beta} \cdot R_{\gamma}$	79.62
Adelmann et al.	$\Re 4$	$R_{TA} \cdot R_{s'} \cdot R_y \cdot R_z \cdot R_{\beta}$	50.68
Brosnihan et al.	$\Re 5$	$R_{TA} \cdot R_{TB} \cdot R_{HE1}$	64.79
Radl	$\Re 6$	$R_m \cdot R_{TB} \cdot R_{HE1} \cdot R_{HE3}$	51.20
Lower Redundant scheme	$\Re 7lr$	$R_X \cdot R_{\alpha} \cdot R_{\beta} \cdot R_{\gamma}$	90.46
Higher Redundant scheme	$\Re 7hr$	$R_{X'} \cdot R_{\alpha} \cdot R_{\beta} \cdot R_{\gamma}$	91.93

$$\Re(t) = e^{-(t/\eta)^{\beta}}\ ,\ h(t) = (\beta/\eta) * (t/\eta)^{\beta-1}$$

$$\Re_s(t) = \sum_{i=0}^{k} \{\int_0^t h(t)dt\}^i e^{-\int_0^t h(t)dt} \cdot (i!)^{-1} = \sum_{i=0}^{k} \frac{(h \cdot t)^i e^{-ht}}{i!}$$

If, 'P' and 'Q' are two components of a system, the series and hot redundancy between components are evaluated by.

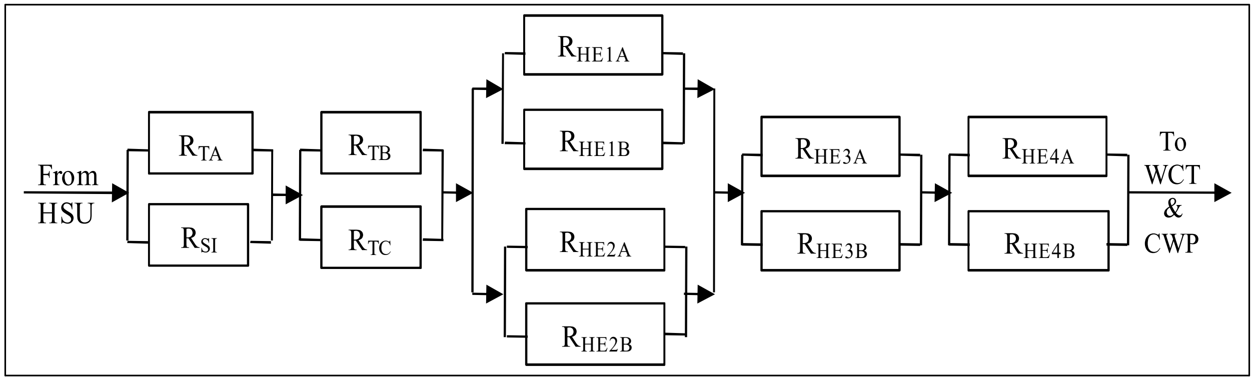
$$\left.\begin{aligned} &\Re_s(t) = 1-(1-R(t))^n \\ &\because \Re_s(t)_{series} = R_P * R_Q \\ &\Re_s(t)_{hot/||} = R_P \,||\, R_Q = 1-\{(1-R_P)*(1-R_Q)\} \end{aligned}\right\}$$

$$h_i(t) = h^{\gamma}\gamma \cdot [\tau(t)]^{\gamma-1} + h_0$$

$$H(t) = [h\,\tau(t)]^{\gamma} + h_0\,\tau(t)$$

- The reliability factors of reservoir $\{0.0-0.9\}$
- The reliability factors of heat exchangers $\{0.0-0.8\}$
- The reliability factors of super insulated vacuum line (SIVL) $\{0.0-0.95\}$

❖ S²HRS-HCS (Six Stage Hot Redundant Structure – HCS)



$$\left.\begin{aligned}
&R_X = 1-\{(1-R_{TA})*(1-R_{SIVL})\}\\
&\therefore R_X = R_{TA} \parallel R_{SIVL}\\
&R_Y = R_{HE1A} \parallel R_{HE1B},\ R_Y = R_Z\\
&\because R_Y = R_{HE1A} \parallel R_{HE1B},\ \ R_Z = R_{HE2A} \parallel R_{HE2B}
\end{aligned}\right\}$$

$$\left.\begin{aligned}
&\because R_S = R_Y * R_{TB},\ R_T = R_Z * R_{TC}\\
&R_\alpha = R_S \parallel R_T,\ \ R_\beta = R_\gamma\\
&\because R_\beta = R_{HE3A} \parallel R_{HE3B},\ R_\gamma = R_{HE4A} \parallel R_{HE4B},\\
&\therefore \Re_{S^2HRS}(t) = R_X * R_\alpha * R_\beta * R_\gamma = \Re_{7lr}(t) = R7lr
\end{aligned}\right\}$$

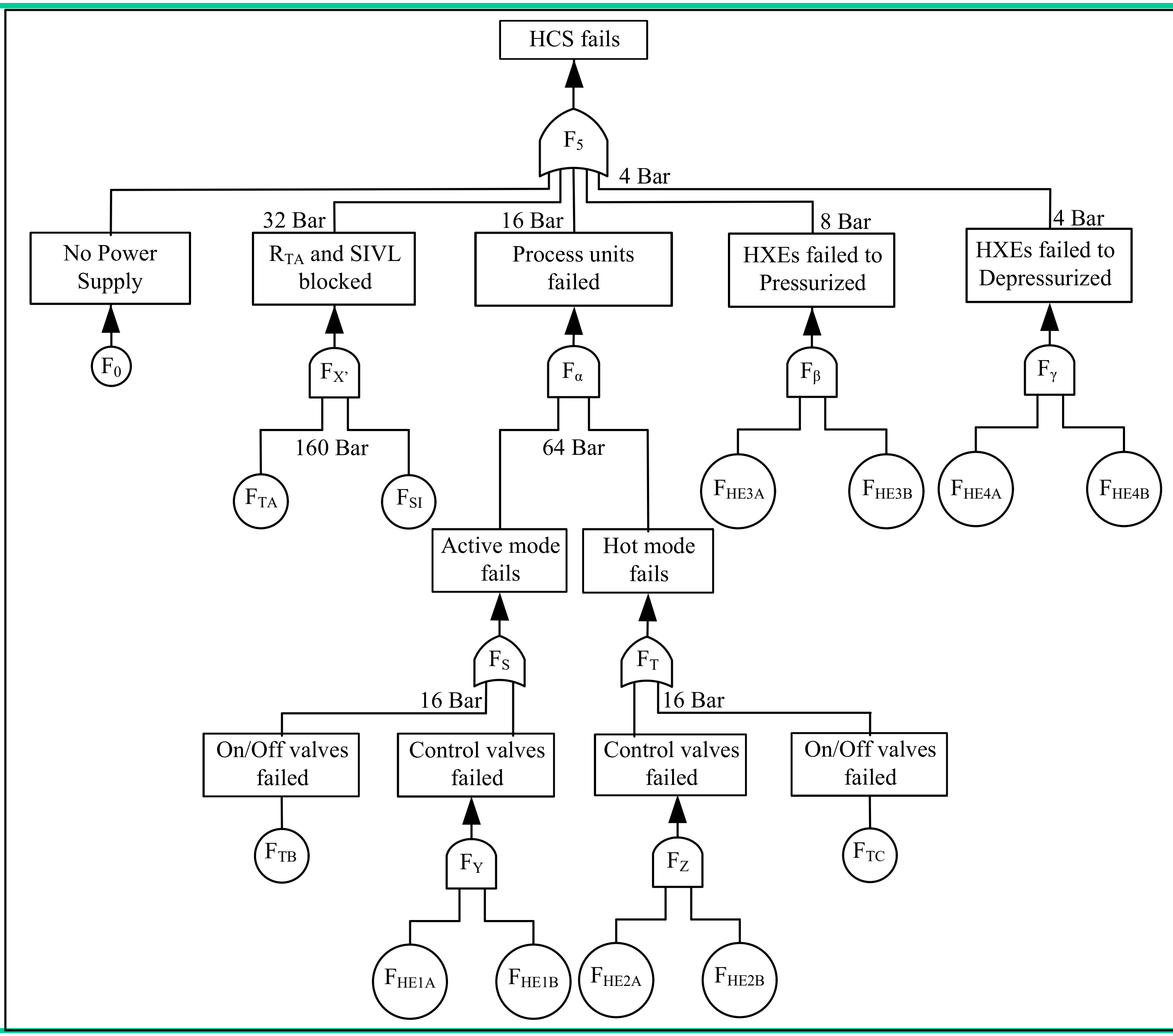
Fault Tree Analysis

- In the work presented, the FTD (fault tree diagram) has been employed to take a lead role of on-line health monitoring of the system by evaluating the performance of the proposed HCS in terms of system failure.

- It has much lesser chances of failure as against chances of failure of the existing systems due to more of if-else conditions. Lesser chances of system failure reflect better efficiency and lesser maintenance requirement on the system.

- Gyan Ranjan Biswal, R. P. Maheshwari, and M. L. Dewal, “System Reliability and Fault Tree Analysis of SeSHRS based Hydrogen Cooling System: Dedicated for Combined Cycle Power Plants,” In Press: *IEEE Systems Journal*, Feb. 2012. DOI: 10.1109/JSYST.2012.2192065

Fault Tree Diagram



$$\left.\begin{array}{l} F_X = F_{TA}\ AND\ F_{SI},\ \ F_Y = F_Z, \\ \because F_Y = F_{HE1A}\ AND\ F_{HE1B}, F_Z = F_{HE2A}\ AND\ F_{HE2B} \\ \because F_S = F_Y\ OR\ F_{TB}, F_T = F_Z\ OR\ F_{TC}, \\ \therefore F_\alpha = F_S\ AND\ F_T, F_\beta = F_{HE3A}\ AND\ F_{HE3B} = F_\gamma \\ \therefore F_{S^2HRS}(t) = F_X\ OR\ F_\alpha\ OR\ F_\beta\ OR\ F_\gamma = F7lr \end{array}\right\}$$

$$\left.\begin{array}{l} \because F_1 = F_X\ OR\ F_\alpha, F_2 = F_1\ OR\ F_\beta, \\ \therefore F_3 = F_2\ OR\ F_\gamma = F_{S^2HRS}(t) = F_{7lr}(t) = F7lr \end{array}\right\}$$

$$F_1 = F_X \; OR \; F_\alpha = F_X + F_\alpha - (F_X * F_\alpha)$$

$$\therefore F_2 = F_1 \; OR \; F_\beta = \{F_X + F_\alpha - (F_X * F_\alpha)\} + F_\beta - \{F_X * F_\beta + F_\alpha * F_\beta - (F_X * F_\alpha * F_\beta)\}$$

$$\therefore F_3 = F_2 \; OR \; F_\gamma = \{F_X + F_\alpha - (F_X * F_\alpha)\} + F_\beta - \{F_X * F_\beta + F_\alpha * F_\beta - (F_X * F_\alpha * F_\beta)\}$$
$$+ F_\gamma - \{F_X * F_\beta * F_\gamma + F_\alpha * F_\beta * F_\gamma - F_X * F_\alpha * F_\beta * F_\gamma\}$$

$$\left.\begin{array}{l} T_R = \eta * \{-\ln[R(T_R)]\}^{1/\beta}, \; if \; R(T_R) = 0.5; \; T_R = \check{T} \\ h(t) = 1 - e^{-(t/MTTR)}, \\ \because \check{T} = -MTTR * \{\ln[1 - h(t)]\} \end{array}\right\}$$

The exact expressions of Tμ=MTTF=MTBF, Ť = T_{med} and $\tilde{T}$ = T_{mod} are as follows:

$$T_\mu = MTTF = MTBF = \eta * \Gamma\left(\frac{1}{\beta} + 1\right) = \int_0^\infty R(t)\, dt$$

$$\check{T} = T_{med} = \eta * (\ln 2)^{1/\beta}, \because h(t) = 1 - e^{-(t/MTTR)}$$

$$\tilde{T} = T_{\text{mod}} = \eta * \{1 - (1/\beta)\}^{1/\beta} \; for \; \beta > 1; \; 0 \; otherwise$$

Thank you